Trajectory generation with rich information content

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Often, trajectories for mechanical systems are generated solving some optimization problem. Common approaches include time-optimal, energy optimal, etc., motion profiles. In order to decrease mechanical wear of real plants this profiles provide, e.g., a smooth movement (rest-to-rest) in accordance with restrictions in jerk, acceleration and velocity. There exists a number of methods, to calculate for a given trajectory the plant feed forward action and to design stabilizing controllers. In case of parameter uncertainty the control law often exhibits some adaptive part. Unfortunately, smooth trajectories tend to contain insufficient excitation for adaption and/or identification. Therefore, we propose to consider some measure for the information content concerning some unknown parameters in the trajectory optimization problem.

1 Introduction

We consider time continuous systems

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)\\ \mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)\end{aligned} \tag{1}$$

with the state vector $\mathbf{x}$, parameter vector $\mathbf{p}$, input vector $\mathbf{u}$ and time t. We use time continuous systems, but evaluate them at certain discrete points of time. Each measurement consist of n samples and the initial guess of unknown parameters $\mathbf{p_0}$ is assumed to fit quite well.

2 Fisher information and optimization problem

The Fisher matrix $\mathbf{F}(\mathbf{u}, \mathbf{p})$ indicates how much information about m unknown system parameters is contained in n samples of a measurable output signal y_r.

$$\mathbf{F}(\mathbf{u}, \mathbf{p}) = \left[\sum_{r=1}^{n_{resp}} \sum_{s=1}^{n_{resp}} \frac{1}{\sigma_{rs}^2} \cdot \mathbf{X}_r^T \cdot \mathbf{X}_s\right] \qquad \mathbf{X}_r = \begin{bmatrix} \frac{\partial y_r}{\partial p_1}\big|_{t_1} & \cdots & \frac{\partial y_r}{\partial p_m}\big|_{t_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_r}{\partial p_1}\big|_{t_n} & \cdots & \frac{\partial y_r}{\partial p_m}\big|_{t_n} \end{bmatrix} \tag{2}$$

$\mathbf{X}_r$ contains the partial derivatives of r-th system output with respect to some parameters evaluated at certain discrete points of time, n_{resp} is the number of measured outputs and σ_{rs}^2 is the rs-th entry of the measurement error covariance matrix. In order to obtain an optimization problem a matrix norm is needed. In literature several criteria for optimizing the fisher matrix are listed [1]. The most common criteria are the determinant (D-criterion) and the trace (A-criterion). A combined constrained dynamic optimization problem can be formulated as following (3) and has to be solved by numerical methods. For now we assume that the initial guess about the unknown parameters is quite good. This assumption can be finally dealt with by an iterative generation and identification scheme.

$$\begin{aligned} &\max_{\mathbf{u}} \|\mathbf{F}(\mathbf{u}, \mathbf{p})\|_{A-criterion}\\ \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)\\ \mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)\\ \mathbf{x}(t_0) &= \mathbf{x_0}\\ \mathbf{x}(t_f) &= \mathbf{x_e}\\ \mathbf{u} &\in \mathbf{U}\\ \mathbf{x} &\in \chi\\ \dot{\mathbf{X}} &= \frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)}{\partial \mathbf{x}} \cdot \mathbf{X} + \frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t)}{\partial \mathbf{p}} \end{aligned} \tag{3}$$

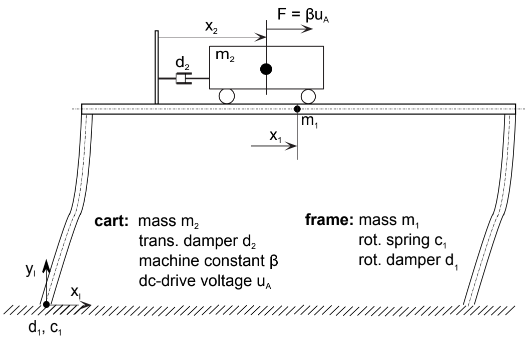

Fig. 1: Cart on flexible frame - schematic

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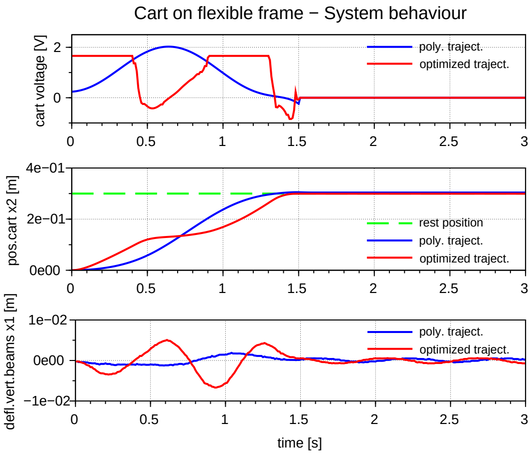

Fig. 2: Time course of system behaviour. Input voltage u_A, cart position x_2 and frame deflection x_1 (red ... optimized trajectory, blue ... polynomial trajectory)

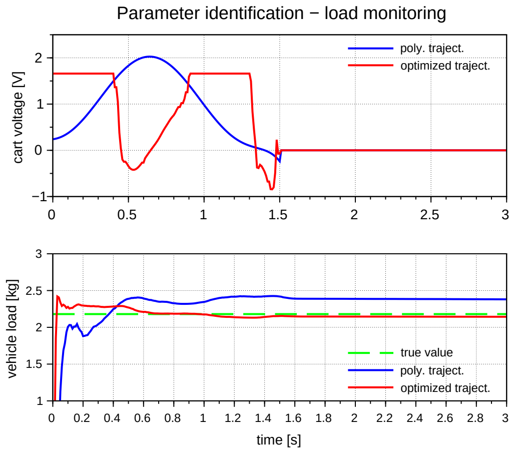

Fig. 3: Load monitoring. Estimation of vehicle load during movement. (red ... optimized trajectory, blue ... polynomial trajectory)

3 2-DOF experiment - cart on flexible frame

As a non trivial real world example, we consider a mechanical 2-degree-of-freedom positioning plant, see Fig.1. The aim is the vibration free positioning of the plant at the rest position and permanent monitoring of the vehicle load for reasons of failure monitoring. The mathematical equations of the system (4) can be computed using Lagrange formalism and can be written in the form $\mathbf{M}(\mathbf{q}) \cdot \ddot{\mathbf{q}} = \mathbf{Q} - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \cdot \dot{\mathbf{q}} - \mathbf{D}(\mathbf{q}, \dot{\mathbf{q}})$ with the vector of generalized coordinates $\mathbf{q} = [x_1, x_2]^T$. For reasons of brevity we assume small displacements of the vertical beams of the frame. Furthermore the static friction is ignored because it's compensated directly.

$$\begin{bmatrix} m_1 + \tilde{m}_2 & \tilde{m}_2 \\ \tilde{m}_2 & \tilde{m}_2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} -c_1 \cdot x_1 - d_1 \cdot \dot{x}_1 \\ \beta \cdot u_A - \tilde{d}_2 \cdot \dot{x}_2 \end{bmatrix} \tag{4}$$

x_2 denotes the position of the cart on the frame and x_1 describes the position of the flexible frame. v_2 and v_1 are the associated velocities. For an accurate parameter estimation an information rich trajectory has to be generated by solving the optimization problem (3) with constraints $\mathbf{x_0} = \mathbf{0}$, $\mathbf{x_e} = [0, 0.3m, 0, 0]^T$, $u_A \in [-1.66V, 1.66V]$, $x_2 \in [0, 0.3m]$ and $v_2 \in [0, 2\frac{m}{s}]$. As mentioned above, the vehicle load $\tilde{m}_2$ has to be identified during the whole movement. For full physical insight into the system parameters, we do not identify the sampled but the continuous plant. To get rid of the time derivatives the second system equation is transformed from time-domain into the frequency-domain. We apply the stable filter with free coefficients $F_0(s) = \frac{\alpha_0}{s^2 + \alpha_1 \cdot s + \alpha_0}$ to the whole ordinary differential equation (ODE) [2]. The inverse Laplace transformation leads to one algebraic equation (AE), which is linear in the unknown parameter.

$$\alpha_0 \cdot \left[\left(x_1 - \frac{\alpha_1}{\alpha_0} \cdot F_1 * x_1 - F_0 * x_1 \right) + \left(x_2 - \frac{\alpha_1}{\alpha_0} \cdot F_1 * x_2 - F_0 * x_2 \right) \right] \cdot \tilde{m}_2 = \beta \cdot F_0 * u_A - \tilde{d}_2 \cdot F_1 * x_2 \tag{5}$$

(* denotes the convolution operator in time-domain) The unknown parameter is estimated using standard recursive least square algorithm. In Fig.2 and Fig.3 real measurement results can be seen. The results are compared to measurements using a standard polynomial trajectory planning approach for rest-to-rest positioning with restrictions in jerk, acceleration and velocity.

4 Conclusion

The performance of the rest-to-rest positioning is quite good at both approaches. The parameter estimation result is appreciably better using the optimized input signal. The load estimation result converges faster to the true value and the stationary estimation error is smaller using the optimized signal.

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References

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